



# Basics of Permanent Magnet Motors and Field Oriented Control

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- Now we'll take a look at basic physics happening in the motor
- It's no rocket science and we will discuss only few equations describing why the motor turns and what's the relation between the current, voltage, momentum, torque and speed
- No more than 10 slides!

## Mechanics

$r$	radius	[m]
$F$	force	[N]
$T$	torque	[Nm]
$T_M$	motor torque	[Nm]
$T_T$	break away (stiction) torque	[Nm]
$T_l$	damping and windage torque	[Nm]
$\tau_{SH}$	shaft instant passive torque	[Nm]
$T_{NET}$	torque on the load	[Nm]
$\omega$	mechanical or electrical speed	[rad·s <sup>-1</sup> ]
$\phi$	mechanical or electrical angle	[rad]

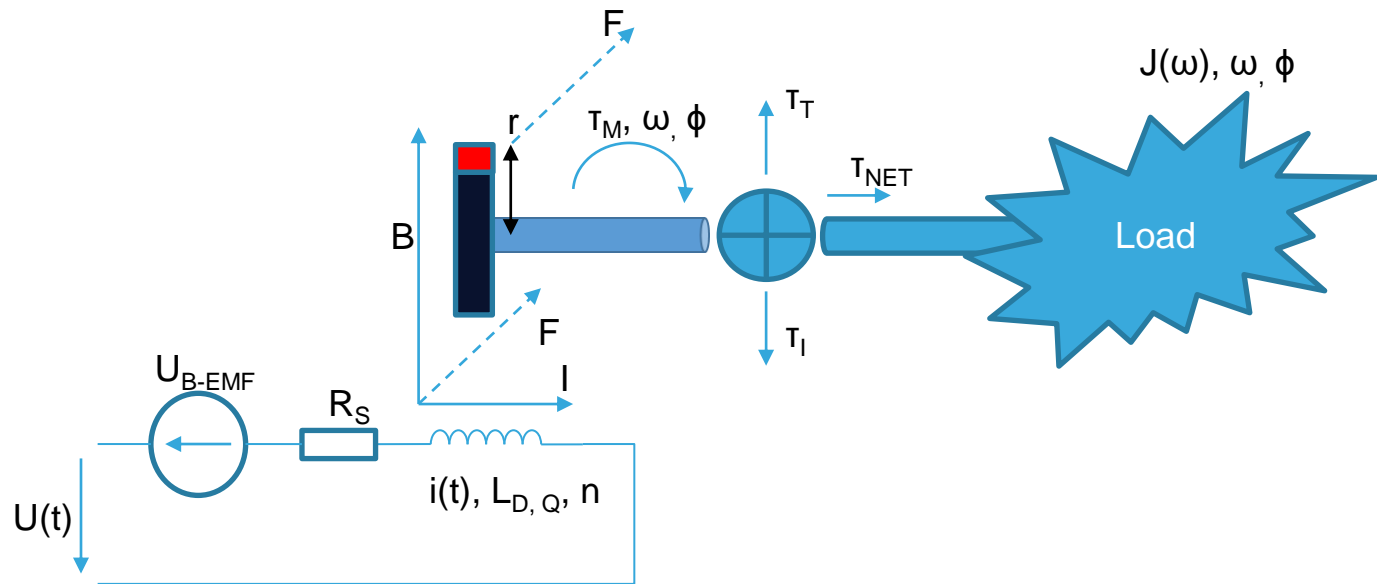
$J$	moment of inertia	[kg·m <sup>2</sup> ]
$D$	damping	[Nms]
$K$	spring constant	[Nm]
$L$	angular momentum	[kg·m <sup>2</sup> ·s <sup>-1</sup> ]
$t$	time	[s]

## Electrics & magnetics

$U, u(t)$	voltage	[V]
$I, i(t)$	el. current	[A]
$R$	el. resistance	[Ω]
$L$	el. inductance	[H]
$n, Z$	number of turns	[-]
$B$	mag. induction	[T]
$\Phi_m$	mag. flux	[Wb]
$l$	loop length	[m]

# What causes rotation & speed? (1/10)

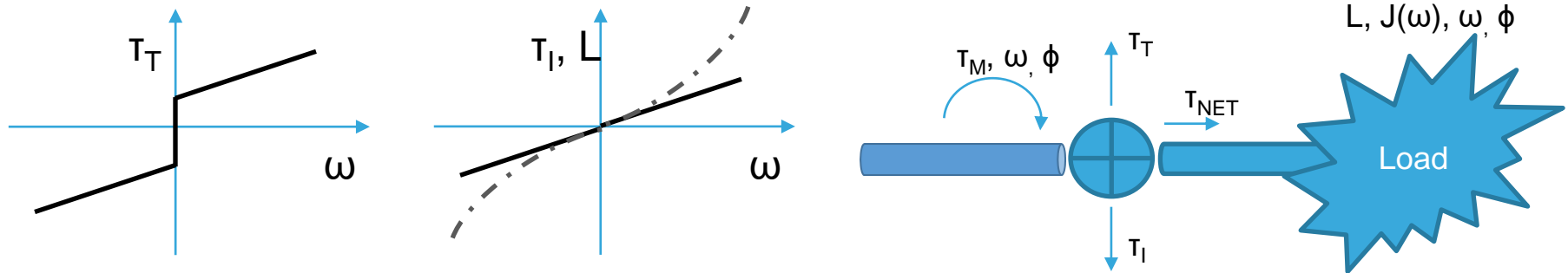
## Our physical system



# What causes rotation & speed? (2/10)

## Mechanics

- The stable torque equation:  $\tau_M + \tau_T + \tau_I + \tau_{NET} = 0$
- $\tau_M$  is motor torque
- $\tau_T, \tau_I$  are mechanical system “resistances”, non-linear

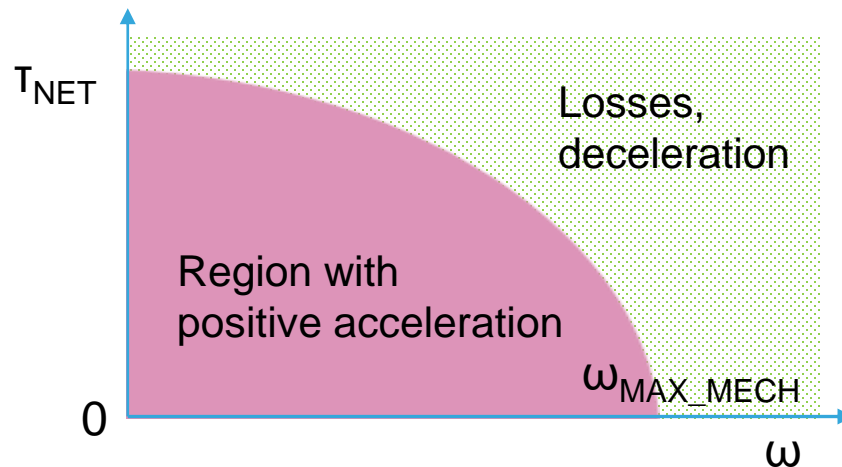


- $\tau_{NET} = J(\omega) \frac{d\omega}{dt}$  is load torque, typically non-linear
- Thus 
$$\omega(t) = \int_{\tau_0}^t \frac{\tau_{NET}(t)}{J(t)} dt + \omega_0$$

# What causes rotation & speed? (3/10)

## Mechanics – wrap up

- Change of speed is equal to available torque:
  - Positive torque remains → motor accelerates
  - Negative torque remains → motor slows down and even reverses
- The bigger the speed, the bigger losses and less torque → acceleration is smaller, down to zero, where we reach maximum speed  $\omega_{\text{MAX\_MECH}}$ :



# What causes rotation & speed? (4/10)

## Mechanics

- Dynamic torque equation (for reference):

$$J \frac{d^2\varphi}{dt^2} + D \frac{d\varphi}{dt} + K\varphi = \pm\tau_M \mp \tau_{SH}$$

J	-	moment of inertia
D	-	damping
K	-	spring constant
$\tau_M$	-	motor torque
$\tau_{SH}$	-	shaft instant passive torque
$\varphi$	-	shaft angle

# What causes rotation & speed? (5/10)

## Mechanics

- Mechanical power on load

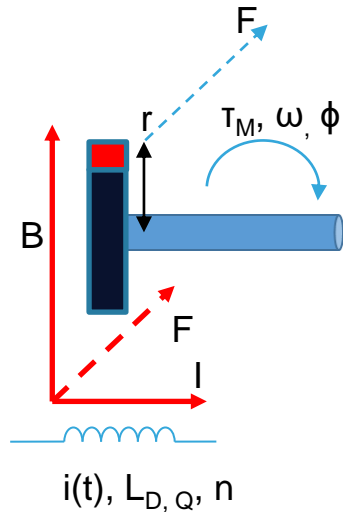
$$P_{MECH.} = \tau_M \omega - D_M \omega^2 - \tau_T \omega$$

$D_M$	-	damping
$\tau_M$	-	motor torque
$\tau_T$	-	resistive torque
$\omega$	-	mechanical speed



# What causes rotation & speed? (6/10)

## Magnetics



- Lorentz law:

$$\vec{F} = I\vec{l} \times \vec{B}, F = IlB \sin \varphi$$

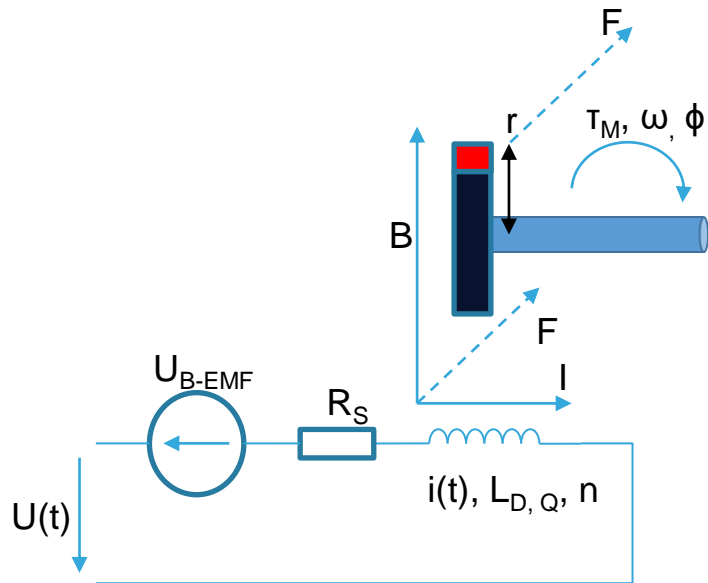
[N; A, m, T]

- $\tau_M = F \cdot r = IlBrn$
- [Nm; N, m, -]

- We control current  $I$  and phase  $\varphi$
- Biggest force (torque) is applied when  $\varphi$  is  $90^\circ$  (i.e.  $\sin(\varphi)=1$ )

# What causes rotation & speed? (7/10)

## Electrics



- $U = L_{D,Q} \frac{dI}{dt} + R_S I + K_e \omega$

$U$  - supply voltage

$K_e \omega$  - b-emf voltage

$$K_e = Brln$$

- electric constant

$R_S I$  - winding losses

$L_{D,Q} \frac{dI}{dt}$  - the useful part

# What causes rotation & speed? (8/10)

## Electrics – wrap up

- Speed is proportional to the voltage
- Torque is proportional to the current
- Torque constant [Nm/A] is proportional to electrical constant [V/krpm]

# What causes rotation & speed? (9/10)

## Electrics – b-emf voltage

- B-emf voltage is induced by rotor turning in the stator winding, with opposite direction of supply current
- For constant supply voltage, the higher the speed, the lower current we can achieve and thus the lower the available torque

$$U = L_{D,Q} \frac{dI}{dt} + R_S I + K_e \omega, \text{ or}$$

$$U = K \cdot \text{torque} + \text{losses} + b\_emf$$

# Effects of B-emf and demagnetization

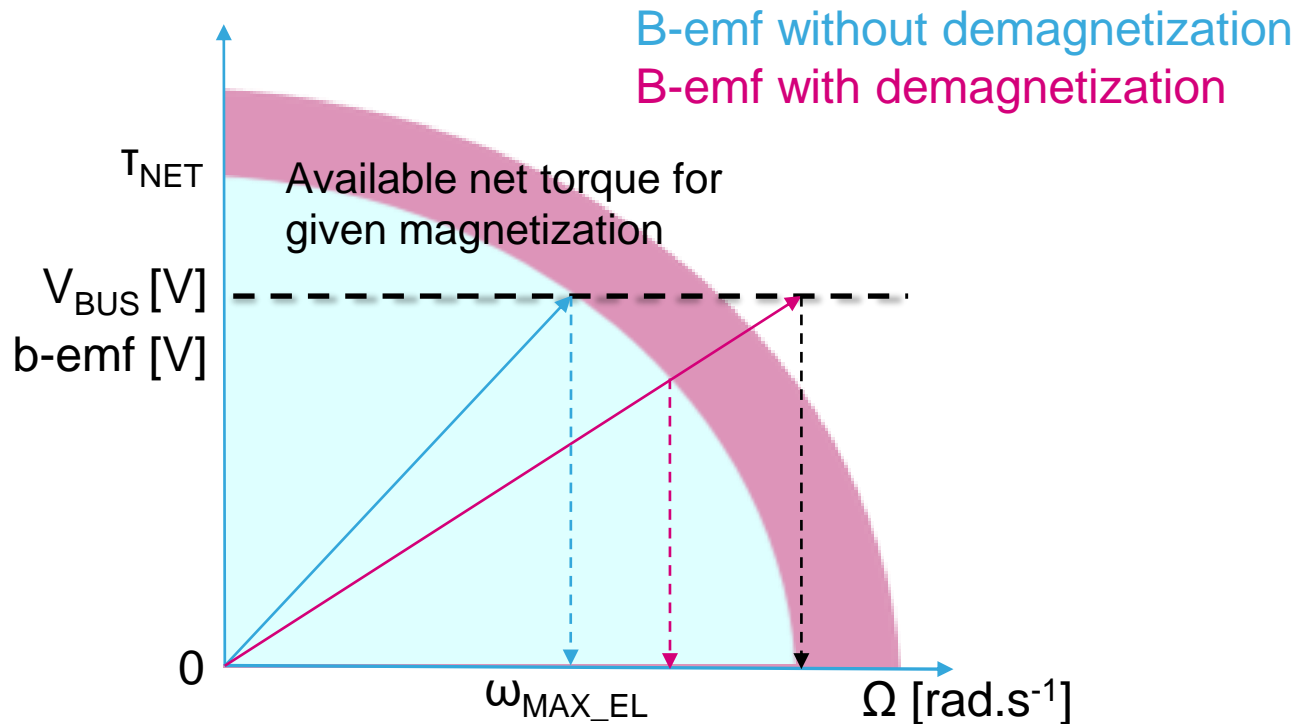
Application of part of coil electromagnetic vector against magnetic field of the permanent magnet in the rotor, which reduces the effective electrical constant  $K' = K - B_D$  and b-emf size

Benefits:

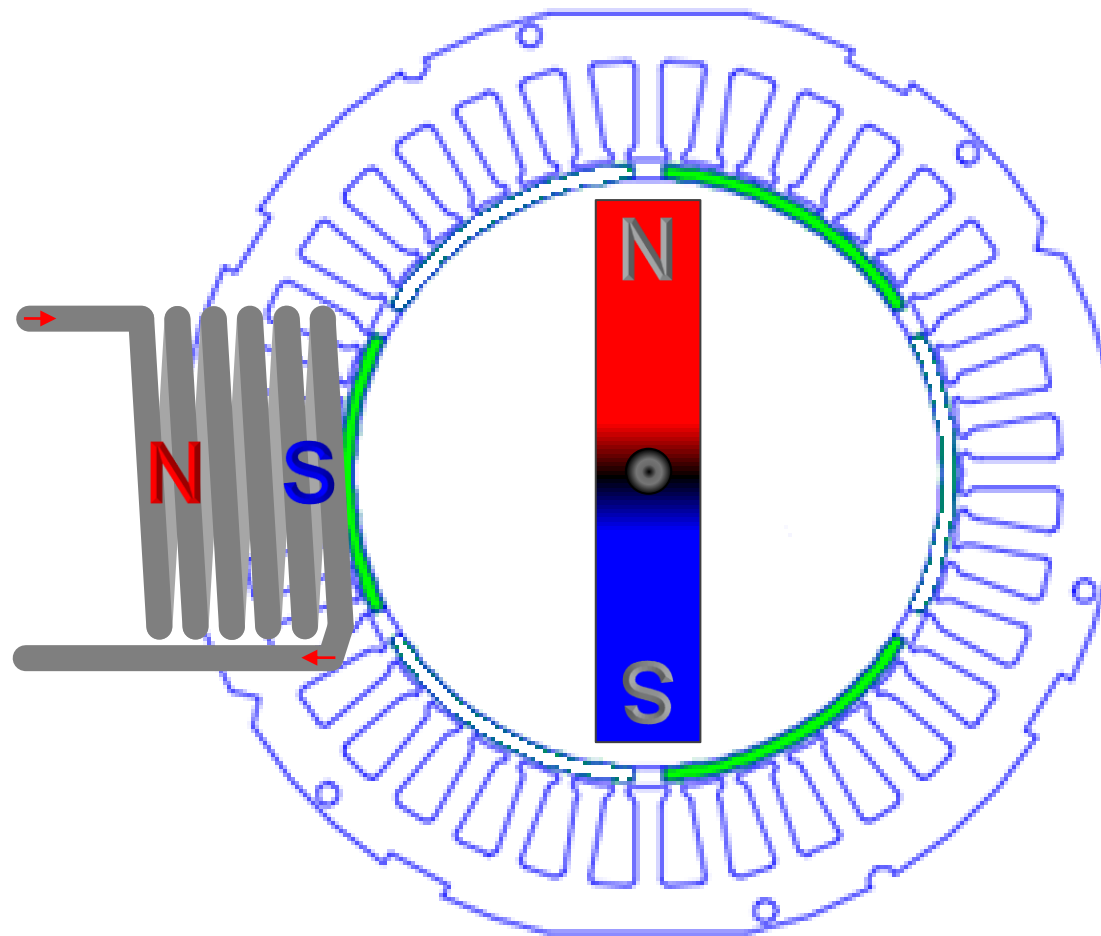
- higher achievable speed
- Smooth speed control

Drawbacks:

- Reduced torque
- Need for overvoltage protection



# PMSM principle – coil & magnetic rotor



**Stator**

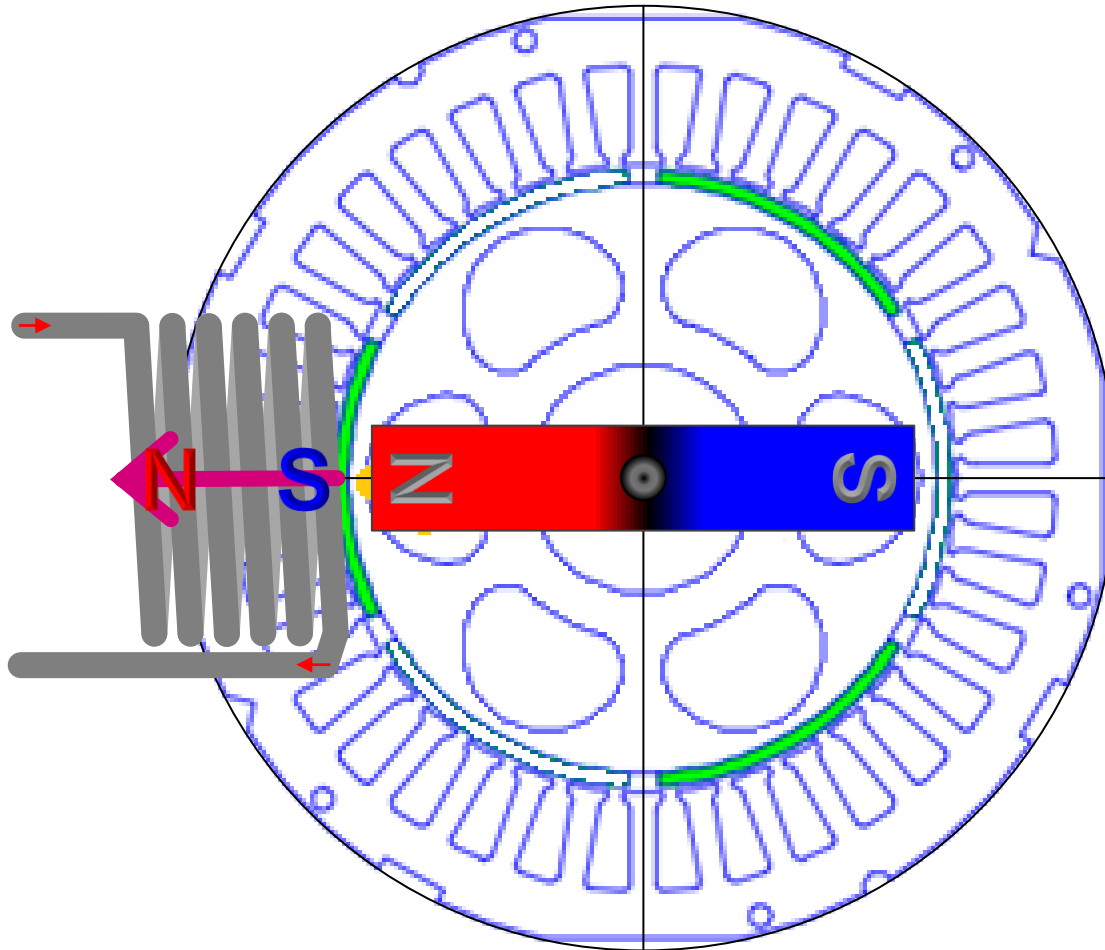
**Rotor**

(permanent magnet) fixed on applicate axis

**Stator coil**

**Current** running in the coil causes rotor to **move** and **align**

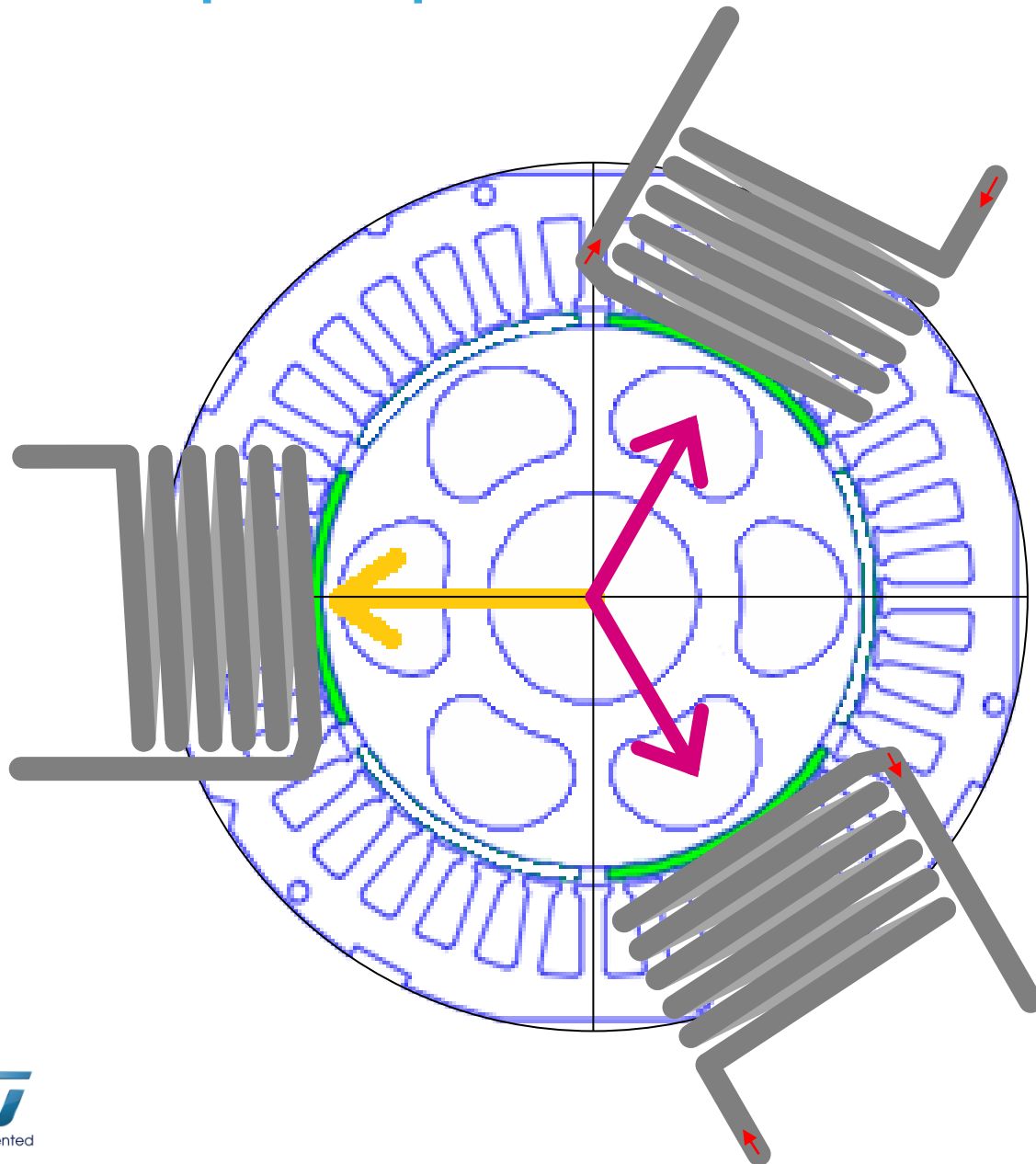
# PMSM principle – vector representation



**Magnetic fields** of the rotor and coil can be described by **vectors** (with their size and direction, or x-y coordinates)

Their **common point** is placed in the **rotor axis**

# PMSM principle – 3 coils – 3 phases

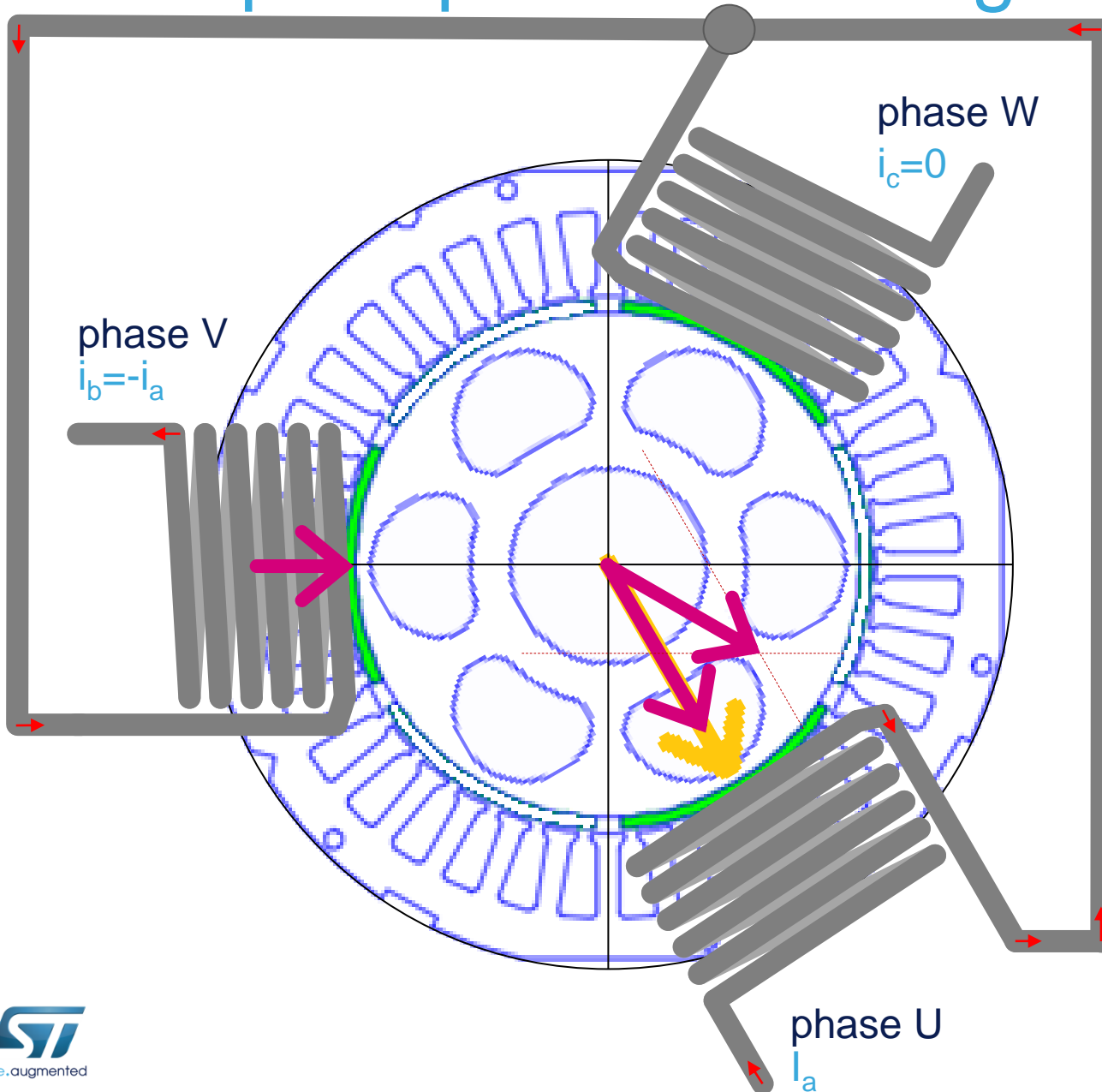


**Second coil**  
Current applied to this coil causes **rotor to move** again

**Third coil**  
Current applied to this coil causes **rotor to move** yet again



# PMSM principle – star configuration

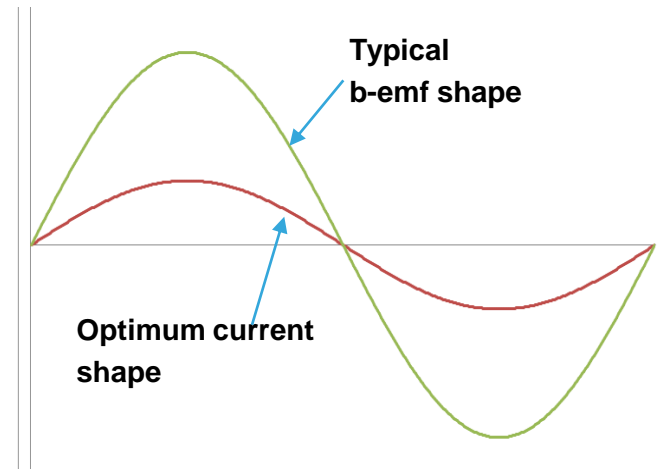
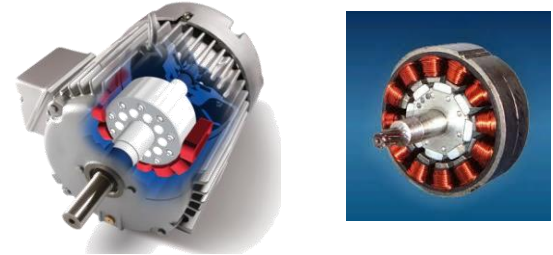


**Stator coils** are connected in the neutral point, only 3 terminals U, V, W are needed

The final stator electromagnetic vector is a sum of those generated by each coil current

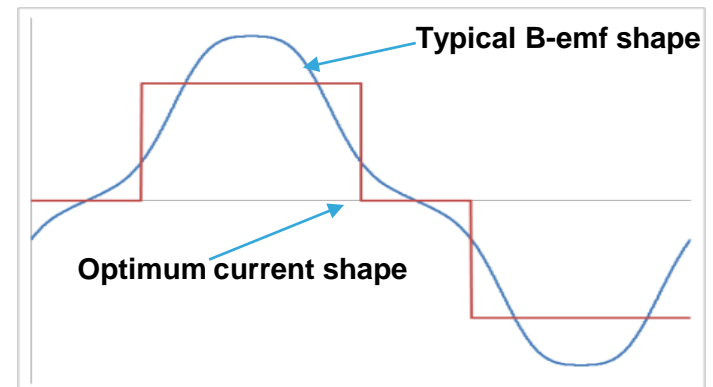
- **Permanent Magnet Synchronous Motor (PMSM)**

- Stator consists of three phase windings
- Rotor houses permanent magnets
  - on the surface → Surface Mounted (SM) PMSM
  - Buried within the rotor → Internal (I) PMSM
- Stator excitation frequency must be synchronous with rotor electrical speed
- Rotation induces sinusoidal Back Electro-Motive Force (B-EMF) in motor phases
- Gives best performances (torque steadiness) when driven by sinusoidal phase current



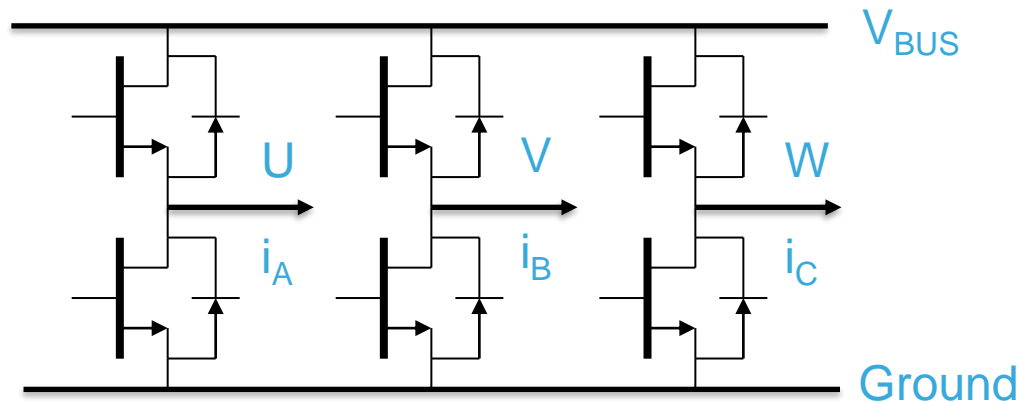
- **Permanent Magnet Brushless DC motors (BLDC)**

- Like PMSM - and despite of their name - require alternating stator current
- Like in PMSM, rotor houses permanent magnets, usually glued on its surface
- Like PMSM, stator excitation frequency matches rotor electrical speed
- Unlike PMSM, rotor spinning induced trapezoidal shaped Back Electro-Motive Force (B-EMF)
- Gives best performances (torque steadiness) when driven by rectangular-shaped currents



# PMSM control principles

- Voltages on phases U, V, W are controlled independently



- 3 currents flow in – or not?

$$i_a + i_b + i_c = 0$$

is equivalent to

$$i_a = -i_b - i_c$$

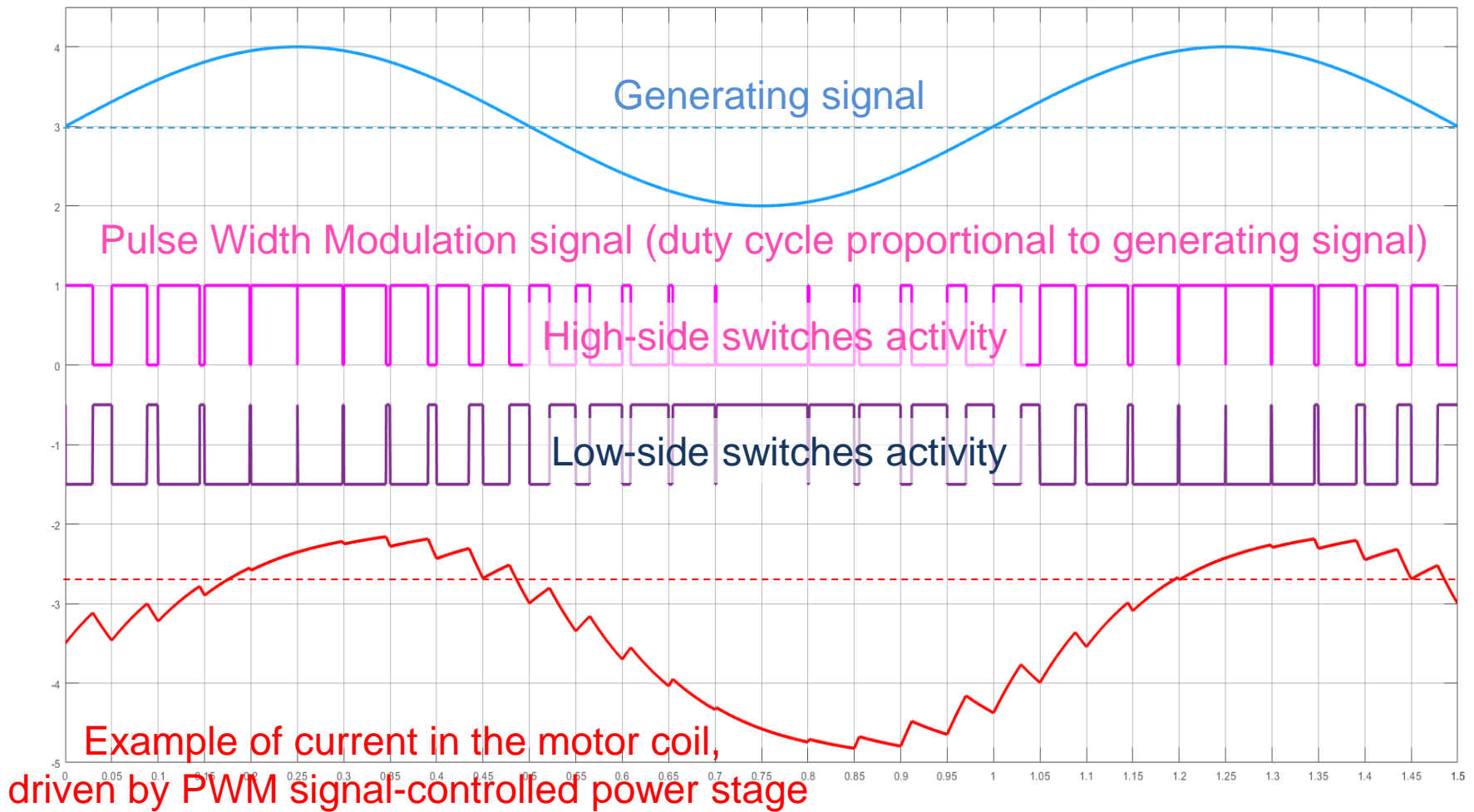
$$i_b = -i_a - i_c$$

$$i_c = -i_a - i_b$$

# PMSM control principles

- All mentioned methods use Pulse Width Modulation with fixed frequency and variable pulse length to control effective voltage on the phases
- Block commutation
  - Historically used
  - Simple to implement
  - Drawbacks in control: higher torque ripple, slow reaction on load change
- Sinusoidal Field Oriented Control (FOC)
  - More complicated and more expensive
  - More difficult to implement, requires DSP-like functionality
  - Rapid reaction on torque/load change, low torque ripple, full 4-quadrant operation

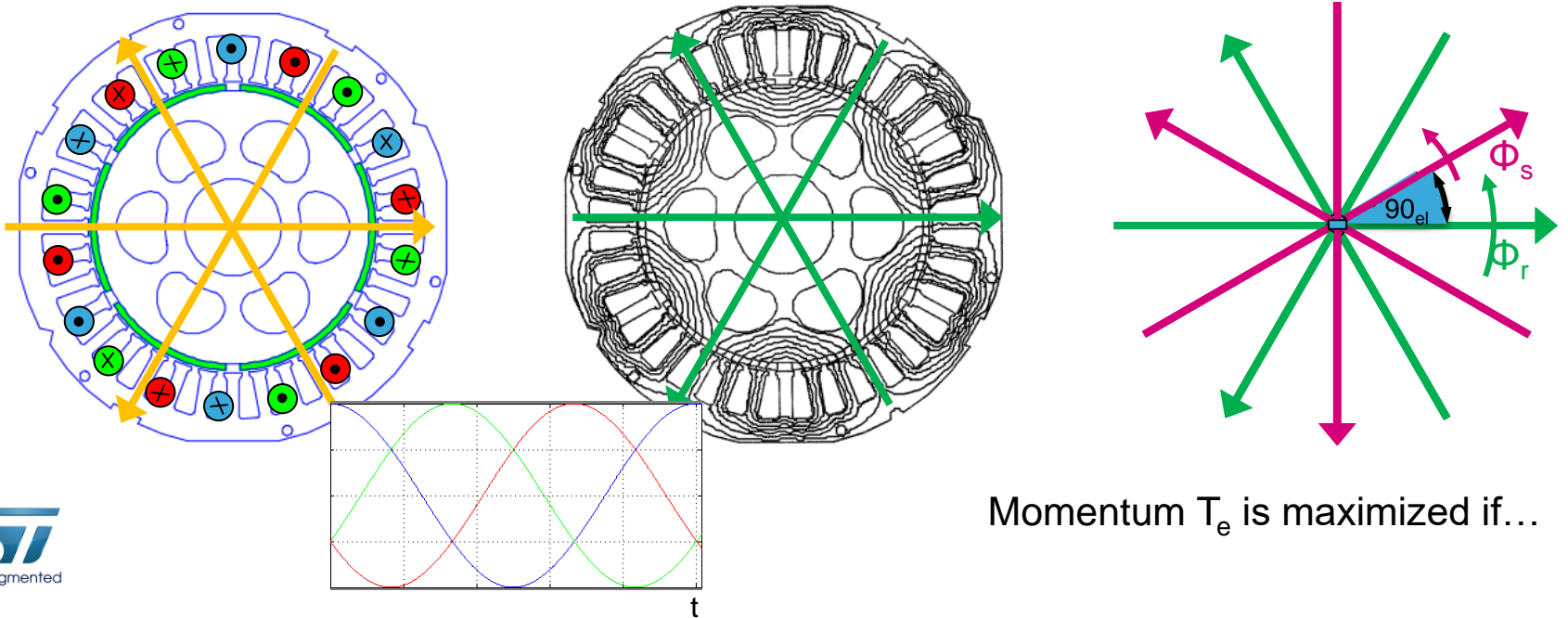
# PWM generation (single phase)



# PMSM FOC principle

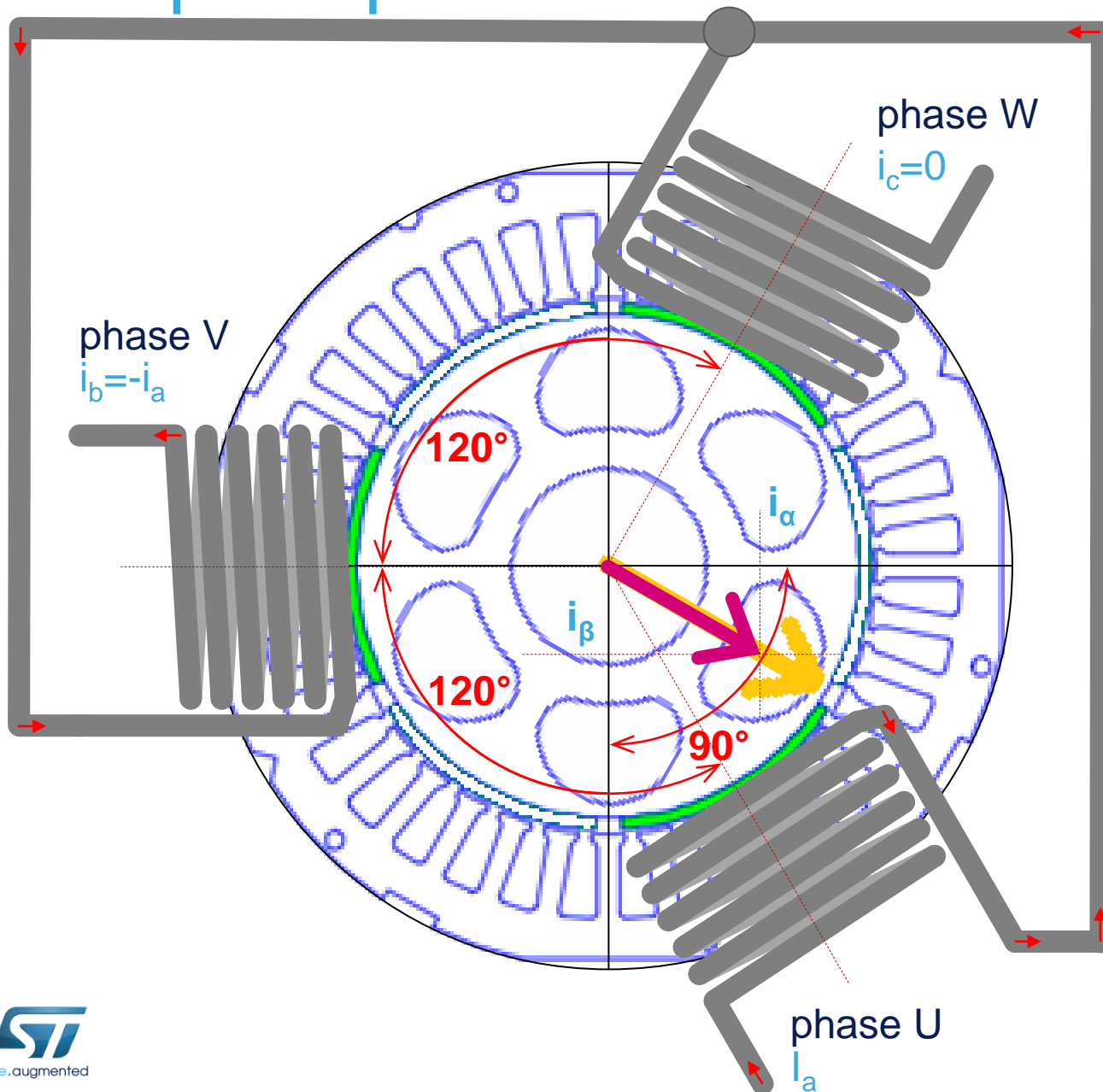
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- Field Oriented Control: stator currents (Field) are controlled in amplitude and phase (Orientation) with respect to the rotor flux
  - current sensing is mandatory (3shunt/1shunt/ICS)
  - speed / position sensing is mandatory (encoder/Hall/sensorless alg)
  - current controllers needed (PI/D,FF)



Momentum  $T_e$  is maximized if...

# FOC principle – Clarke transformation



Clarke:

transforms

$i_a, i_b, i_c$  ( $120^\circ$ )

to

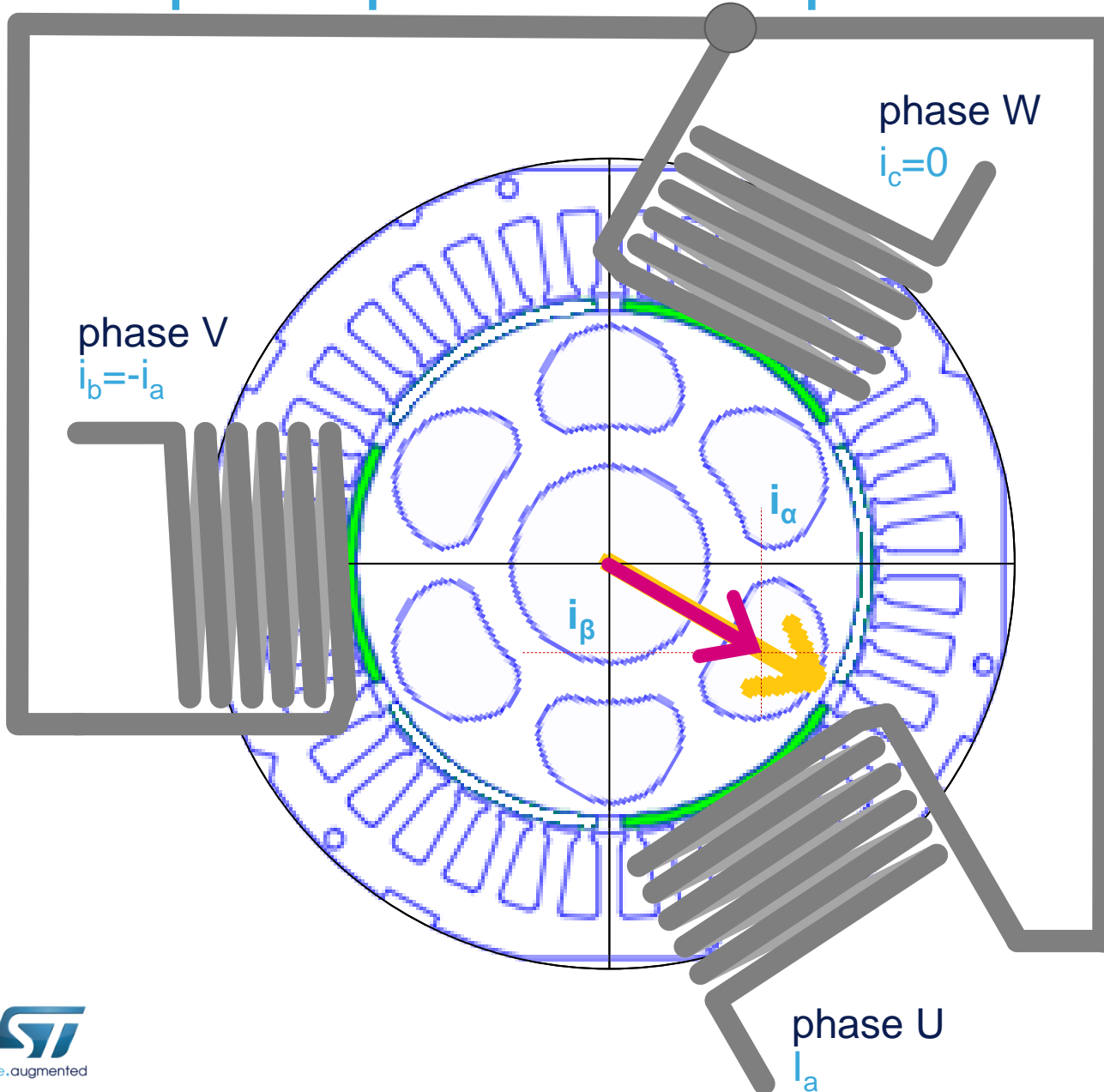
$i_\alpha, i_\beta$  ( $90^\circ$ )

considering

$$i_a + i_b + i_c = 0$$



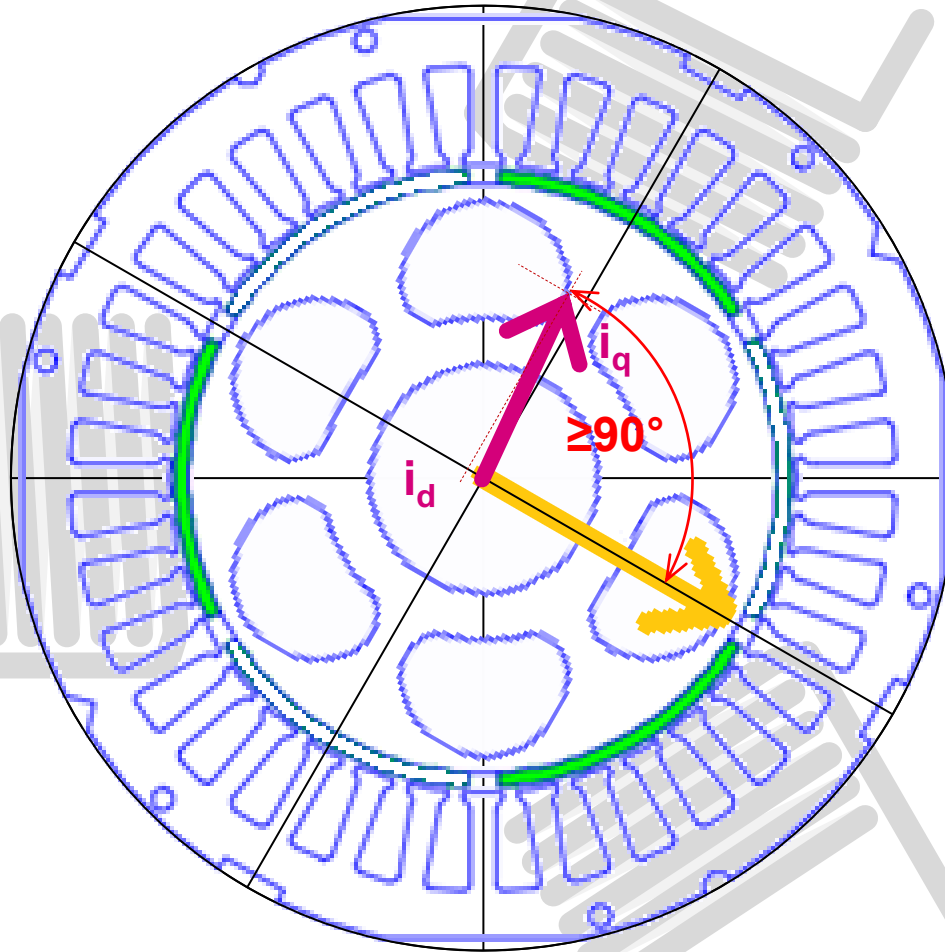
# FOC principle – axes spin with the rotor



Lock of axial system to rotor position

Control mechanism stabilizes angle difference between rotor and stator fields close to  $90^\circ$  (or a little more if MTPA algorithm is used)

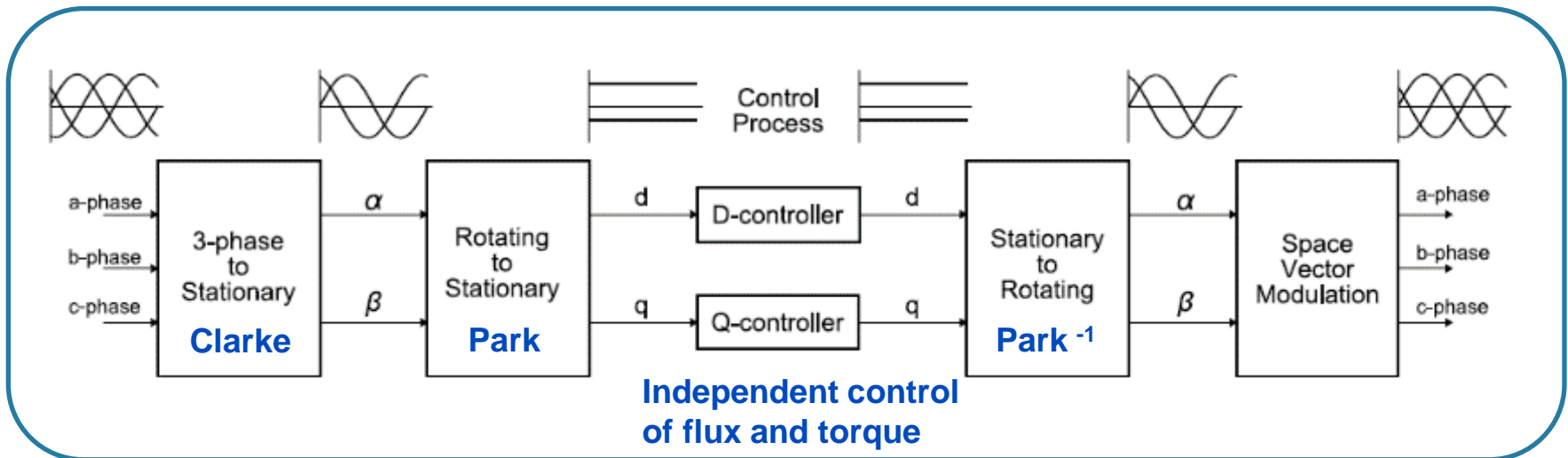
# FOC principle – Park transformation



Park:

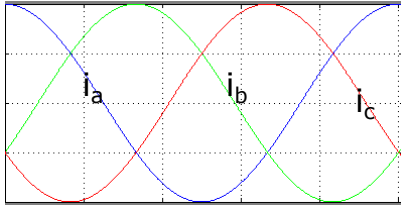
transforms  
rotating  $i_\alpha$ ,  $i_\beta$   
to steady  
values  $i_q$ ,  $i_d$   
seen from the  
rotor view

- Best energy **efficiency** even during **transient operation**.
- **Responsive speed control** to load variations.
- **Decoupled control** of both electromagnetic torque and flux.
- Acoustical **noise reduction** due to sinusoidal waveforms.
- Active **electrical brake** and **energy reversal**.



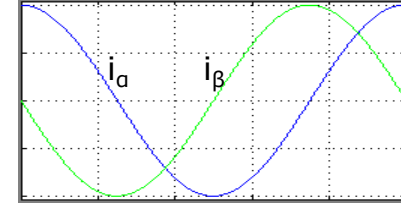
# PMSM FOC overview: reference frame transformations

- Clarke: transforms  $i_a, i_b, i_c$  ( $120^\circ$ ) to  $i_\alpha, i_\beta$  ( $90^\circ$ )

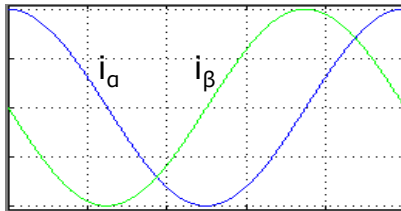


$$i_\alpha = i_{as}$$

$$i_\beta = -\frac{i_{as} + 2i_{bs}}{\sqrt{3}}$$

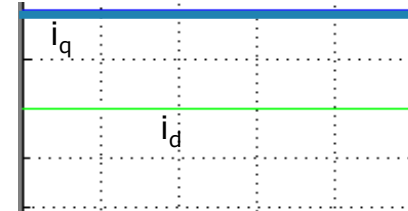


- Park: currents  $i_\alpha, i_\beta$ , transformed on a reference frame rotating with their frequency, become DC currents  $i_q, i_d$  ( $90^\circ$ ) – a demodulation

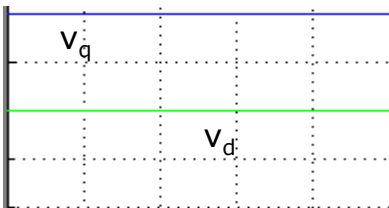


$$i_{qs} = i_\alpha \cos \theta_r - i_\beta \sin \theta_r$$

$$i_{ds} = i_\alpha \sin \theta_r + i_\beta \cos \theta_r$$



- PI regulators now work efficiently in a 'DC' domain; their DC outputs, voltage references  $v_q, v_d$  are handled by the Reverse Park into AC domain ( $v_\alpha, v_\beta$ )



$$v_\alpha = v_{qs} \cos \theta_r + v_{ds} \sin \theta_r$$

$$v_\beta = -v_{qs} \sin \theta_r + v_{ds} \cos \theta_r$$

